Area $DEG=\pi r^2(1-\sin\theta\operatorname{cosec}\phi)^2$.

$u=x^2(\phi\operatorname{cosec}^2\phi-\cot\phi)=r^2\sin^2\theta(\phi\operatorname{cosec}^2\phi-\cot\phi)$.

An element of surface at Q is $4xdxd\rho=4r^2\sin\theta\cos\theta d\theta d\rho$; at R it is $du=2r^2\sin^2\theta\operatorname{cosec}^2\phi(1-\phi\cot\phi)d\phi$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of ϕ, θ and $\pi-\theta$; of ρ, 0 and 2π.

Hence, doubling as R may be on either side of PQ, we get for the required chance,

$$p=\frac{16\pi r^6}{\pi^3 r^6}\int_0^{\frac{1}{2}\pi}\int_\theta^{\pi-\theta}\int_0^{2\pi}\sin^3\theta\cos\theta\operatorname{cosec}^2\phi(1-\phi\cot\phi)(1-\sin\theta\operatorname{cosec}\phi)^2 d\theta d\phi d\rho$$

$$=\frac{32}{\pi}\int_0^{\frac{1}{2}\pi}\int_\theta^{\pi-\theta}\sin^3\theta\cos\theta\operatorname{cosec}^2\phi(1-\phi\cot\phi)(1-\sin\theta\operatorname{cosec}\phi)^2 d\theta d\phi$$

$$=\frac{2}{3\pi}\int_0^{\frac{1}{2}\pi}[2(\pi-2\theta)\sin2\theta+4-4\cos4\theta-3\sin^2 2\theta\cos2\theta$$

$$+64\sin^4\theta\cos\theta\log\tan\tfrac{1}{2}\theta]d\theta=\tfrac{2}{5}.$$

Also solved by F. P. Matz.

Miscellaneous 146 was also solved by Jeannette Brooks.

PROBLEMS FOR SOLUTION.

ALGEBRA.

223. Proposed by THEODORE L. DELAND, Office of the Secretary of the Treasury, Washington, D.C.

An officer in the Treasury Department assigned three clerks to count a lot of silver dollars and when finished noted that there was an apparent difference in their efficiency; and, to determine the fact, gave to each a similar lot of the same amount to count, the only record made at the time being that A to count his lot alone, took three weeks longer, B took two weeks longer, and C took one week longer than it took for all working together to count the first lot. The best counter, on the record made, was given an efficiency mark of 93 on the scale of 100. What efficiency mark should, on the record, be given to each of the other two counters?

224. Proposed by G. W. GREENWOOD, M. A. (Oxon). Lebanon, Ill.

Show that, if none of the quantities x, y, z is zero, the result of eliminating them from $(x+y)(x+z)=bcyz$ ....(1),

$(y+z)(y+x)=cazx$ ....(2),

$(z+x)(z+y)=abxy$ ....(3),

is $\begin{vmatrix} \pm a, & 1, & 1 \\ 1, & \pm b, & 1 \\ 1, & 1, & \pm c \end{vmatrix}=0.$

[Oxford, 1896.]

225. Proposed by H. M. ARMSTRONG, Cooch's Bridge, Delaware.

If $\alpha=ax+cy+bz$ ....(1), $\beta=cx+by+az$ .... (2), $\gamma=bx+ay+cz$......... (3), show that $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma=(a^3+b^2+c^3-3abc)(x^3+y^3+z^3-3xyz)$.

GEOMETRY.

248. Proposed by CHRISTIAN HORNUNG, Heidelberg University, Tiffin, Ohio.

Given AB, BC in a straight line, to produce it to D so that $AD.CD=BD^2$.

249. Proposed by W. W. BEMAN, The University of Michigan.

Given the distances of a point in the plane of a square from three of its vertices, to find the side of the square.

250. Proposed by W. W. BEMAN, The University of Michigan.

Given the distances of a point in the plane of an equilateral triangle from the vertices; to find the side of triangle. [Perkins' *Geometry*, Olney's *Geometry*.]

CALCULUS.

189. Proposed by J. E. SANDERS, Hackney, Ohio.

Solve $d^2y/dx^2=-\beta^2(p+y)$, p and β being constants. The initial conditions are $y=0$ for $x=0$, l; $dy/dx=0$ for $x=l/2$. [Merriman's *Mechanics*, 9th Ed., 1903, §62.]

190. Proposed by SAUL EPSTEEN, The University of Chicago, Chicago, Ill.

$$\int_0^\infty \frac{\sin x\cos\beta x}{x}dx,\ \int_0^\infty \frac{\sin\alpha x\cos x}{x}.$$

MECHANICS.

170. Proposed by M. E. GRÁBER, A. M., Heidelberg University, Tiffin, Ohio.

Prove that the moment of inertia of an ogival head rotating about its geometrical axis is $\frac{\pi w}{g}\int_0^{R\sqrt{(4n-1)}} y^4dx$, where w is the weight in pounds of a cubic foot of material, R the radius of the base of the ogive, and n the diameter of projectile.

DIOPHANTINE ANALYSIS.

123. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

Of two numbers $a_ib_ic_id_ie_i$ $(i=1, 2)$ it is given that their 10 digits a_1, ..., e_2 form a permutation of 0, 1, ..., 9, and that the sum of the two is $x3951$. Give an immediate evaluation of x; also list the possible pairs a_1, a_2; .....; e_1, e_2.